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# Time, Quality and Growth

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## Time, Quality and Growth

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#### Abstract

Consumption requires time (consumption and time are complements of each other). In addition higherquality goods provide more utility per unit of time allocated to consumption, though at a higher monetary cost. Since time is limited, higher income is decreasingly spent augmenting the number of units of goods being consumed and increasingly spent upgrading their quality. After analyzing the basic microeconomics of consumer quantity/quality choices, this working paper investigates its implications on growth. As a country develops, raising the quality of output becomes increasingly important as a component of gross domestic product (GDP) growth relative to quantity growth. Furthermore technological progress is increasingly quality-biased. Lower income inequality raises the scale of output while reducing average quality. This is positive for technical progress and growth at early stages of economic development but may be negative at later stages. These results are broadly consistent with the existing empirical evidence on the composition of GDP growth, international trade patterns of vertical specialization across countries, and the nonlinearity of the impact of inequality on growth. This working paper also explores the potential role of progressive consumption taxes as a growth policy.

#### Key words

Allocation of time, product quality, inequality, growth, distortionary consumption taxes.

#### Resumen

El consumo requiere tiempo (tiempo y consumo son bienes complementarios). Además, los bienes de mayor calidad dan más utilidad por unidad de tiempo asignada al consumo, aunque a mayor coste. Como el tiempo es limitado, los aumentos de la renta se gastan decrecientemente en aumentar el número de unidades consumidas v de manera creciente en aumentar la calidad de los bienes consumidos. Tras analizar la microeconomía de estas decisiones del consumidor sobre cantidad versus calidad, se investigan sus implicaciones sobre el crecimiento. Según se desarrolla un país, el aumento de la calidad de la producción constituye un elemento crecientemente importante del incremento del producto interior bruto (PIB) respecto al crecimiento de la cantidad. Además, el progreso tecnológico está sesgado hacia la producción de mayor calidad. Una menor desigualdad en la distribución de la renta aumenta la producción y reduce su calidad. Esto es positivo para el progreso tecnológico y el crecimiento en las etapas iniciales del desarrollo económico, pero puede ser negativo en fases más avanzadas. Estos resultados son consistentes con la evidencia empírica existente sobre la composición del aumento del PIB. los patrones de especialización internacional según la calidad, y la no linealidad del efecto de la desigualdad sobre el crecimiento. También se explora el papel que pueden jugar los impuestos indirectos progresivos en la política de estímulo al crecimiento.

#### Palabras clave

Asignación del tiempo, calidad del producto, desigualdad, crecimiento, impuestos indirectos progresivos. Al publicar el presente documento de trabajo, la Fundación BBVA no asume responsabilidad alguna sobre su contenido ni sobre la inclusión en el mismo de documentos o información complementaria facilitada por los autores.

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#### 1. Introduction

FOR most goods, production and expenditure can be seen as involving two dimensions: quantity and quality. For example, firms may increase the production of jackets by increasing their number or by raising their quality; consumers may increase their expenditure in restaurants by dining out more often or by going to better restaurants, etc. Most macroeconomic models do not distinguish between these two dimensions. However the distinction is increasingly important as suggested by the facts discussed further. This working paper provides a new approach to the analysis of consumer quantity/quality choices and explores their implications for growth. The working paper considers four issues in a very tractable framework: the increasing importance of quality growth as a component of gross domestic product (GDP) growth; the quality bias of technological progress; the potential of progressive consumption taxes as a growth policy; and the effect of inequality on growth through the quantity/quality channel.

The relevance of the quantity/quality distinction is becoming apparent in a number of areas, even if measuring quality is admittedly a very difficult endeavor. There is increasing evidence of the importance of quality growth as a component of GDP growth. For example, Bils and Klenow (2001) estimate that annual quality growth averaged 3.7% for consumer durable goods in the U.S. over 1980-1996 (about 60% of this growth would be wrongly accounted for as inflation by the Bureau of Labor Statistics [BLS]). In turn, Bils (2005, 2009) estimates an annual quality growth for durables of about 5% since 1988. So far, thorough estimates of the quality growth component of GDP growth are not available, but some calculations suggest that it could account for a very large portion of GDP growth. Moulton and Moses (1997) use data for 1995 and estimate that BLS methods may account for as much as one percent quality growth that year. To this we should add the unmeasured portion of quality growth. The Boskin Commission Report (1996) argued that unmeasured quality change was the most important source of consumer price index (CPI) upwards bias in the U.S. and would be responsible for an approximate effect of 0.6% per year  $^1$ .

<sup>1.</sup> The literature on the upward bias in the measurement of CPI inflation due to unmeasured quality growth is extensive. For example, Gordon (2006) estimates that, even after the thorough

This importance of quality growth as a component of GDP growth seems to have increased over time, since backwards extrapolation of recent estimations of quality growth and CPI bias would result in very improbable low levels of consumption in previous centuries <sup>2</sup>.

The recent international trade literature also emphasizes the quality dimension in describing the current patterns of international trade and provides evidence of the increasing importance of the quantity/quality distinction. On the demand side, trade is characterized by richer countries importing relatively more from countries that produce higher-quality goods (Hallak, 2006). Along the same line, country pairs with similar income distributions have similar import price distributions, suggesting similar quality distributions of imports (Choi, Hummels and Xiang, 2009). On the supply side, horizontal country specialization across goods is losing importance relative to vertical (quality) specialization within goods. Both low and high wage countries are increasingly exporting the same kinds of goods though of different average qualities. Richer countries tend to export higher quality (Schott, 2004; Hummels and Klenow, 2005; Fontagne, Guillaume and Zignago, 2008). A possible explanation for the comparative advantage of richer countries in producing higher-quality goods is that these goods tend to be more intensive in physical and human capital. Complementary to this explanation is that richer countries use more advanced technologies, and that technical progress tends to be quality biased. The model in this working paper is consistent with all these facts.

The starting argument of the analysis can be stated as follows. Consumption requires time. Individuals need time to listen to a concert, travel for pleasure, or play with a game console. Since time is limited, this reduces the possibility of increasing utility by increasing the quantity of consumption (i.e., by increasing the number of units of the goods being consumed). In turn, higher-quality goods provide higher utility per unit of time allocated to consumption, though at a higher monetary cost. The consequence of

methodological revisions recently put in place by the BLS, there is still an upward bias in the measurement of CPI annual inflation of at least one percentage point, which is due to a large extent to unmeasured quality upgrading. See also Shapiro and Wilcox (1996) and Gordon (2005); and Bils and Klenow (2001) and Bils (2005) for further references.

<sup>2.</sup> According to Gordon (2005): "While the CPI may have overstated inflation in the mid-1990s by about 1% per year, as concluded by the Boskin Commission, it does not make sense to extrapolate that rate of bias backwards over long periods of time. The *Hulten-Bruegel paradox* shows that any such exercise in backward extrapolation yields levels of real consumption two or four centuries ago that are implausibly low, barely providing an average household with a pound of potatoes per day, with nothing left over for clothing or shelter."

these circumstances is that higher income is decreasingly spent augmenting the number of units of goods being consumed and increasingly spent upgrading their quality <sup>3,4</sup>. Most models considering the quality dimension of goods typically assume some sort of positive relationship between income and the demand for quality. However this positive relationship is taken as a fact without further exploration of possible underlying reasons. The analysis in this working paper shows that the positive relationship between income and the demand for quality can be explained as a consequence of the time constraint and the complementarity between consumption and time. The literature has paid little attention to the implications of the complementarity between consumption and time in spite of the early work by Becker (1965) <sup>5</sup>. And, in particular, the connection between the time constraint and the demand for higher-quality goods seems to be completely neglected.

Our working paper first analyzes the basics of consumer quantity/quality decisions along the lines just sketched. This analysis is then embedded into a growth model. The natural growth implication of the consumer analysis is that, as a country develops, raising the quality of output becomes increasingly important as a component of GDP growth, relative to quantity growth. This is consistent with the empirical evidence of the large importance of quality growth in recent times cited above, and the unlikeliness that these estimates could be extrapolated backwards. Efficiency in the model has two dimensions, each linked to one of the two dimensions of production: quantity and quality. Endogenous growth is introduced by assuming learning-by-doing in each of these two dimensions. The model predicts that technological progress is increasingly biased in favor of reducing the pro-

<sup>3.</sup> In other words, consider both the time and monetary costs of obtaining utility through consumption. Since time has a higher monetary value to richer individuals, and quality saves time to obtain utility, consuming higher-quality goods provides relatively cheaper utility to richer individuals. Hence richer individuals consume higher quality.

<sup>4.</sup> Some durable goods such as washing machines and vacuum cleaners save time for individuals. However these goods are better seen as capital goods used for household production instead of as consumption goods. See, for example, the theoretical and quantitative analysis in Greenwood, Seshadri and Yorukoglu (2005). The concept of household production was introduced by Becker (1965). The allocation of time to different types of consumption, leisure, and home production across the life cycle is investigated by Mark Aguair and Erik Hurst in a series of papers (see, for example, Aguair and Hurst, 2007 and Aguair and Hurst, 2008).

<sup>5.</sup> There are important exceptions, however. Goolsbee and Klenow (2006) analyze consumer decisions with respect to goods whose main cost is the time spent on their consumption (e.g., watching TV and using the Internet for fun). In turn, Hall and Jones (2007) show that if the marginal utility of consumption is decreasing, consumers will spend an increasing share of income on extending life expectancy, as they get richer. The underlying reason is similar to the starting point in our working paper: individuals need time to obtain utility from consuming more goods.

duction costs of higher-quality goods. Again, this is consistent with the evidence showing that richer countries have a comparative advantage in producing higher-quality goods. It also shows that the model can be calibrated to produce reasonable long-run paths of GDP growth. Nonetheless the simple endogenous growth model built in this working paper seems to strongly overestimate the quality component of GDP growth. Thus an important task for future research is to build and calibrate more flexible models that can do a better job at matching the quantity and quality components of GDP growth, as more thorough estimates of these components will hopefully become available.

The quantity/quality composition of output affects the intensity and quality bias of technical progress and, therefore, affects growth. This quantity/quality composition is determined by consumer choices which can be influenced by progressive consumption taxes. As a result, progressive consumption taxes can be used as a growth policy. In the model, the key constraint to technical progress at early stages of development is the small scale of production. Both learning in the quantity and the quality dimensions of efficiency require a large output. As a consequence, growth can be enhanced by shifting the quantity/quality composition of GDP in favor of a larger scale. Therefore growth can be heightened at early stages of development by charging higher taxes on higher-quality goods. However this initially positive impact of progressive consumption taxes on growth can change its sign at later stages of development. The intuition is that at later stages of development, output has already reached a large scale, and quality upgrading is becoming the most important goal of consumers.

Inequality also affects growth by means of a similar mechanism. Since preferences are nonhomothetic with respect to quantity/quality pairs, income distribution affects the composition of output. Hence inequality affects growth through its influence on the quantity/quality composition of output, which in turn affects the intensity and quality bias of technical progress. This mechanism is the last issue explored in the working paper. The model predicts that higher inequality hinders growth at early stages of development. The reason is that higher inequality gives rise to higher average quality of GDP but smaller volume of output. And, as already noted, reaching a large scale of output is more important than producing high quality in order to foster technical progress at early stages of development. Nonetheless, again, the sign of this inequality-growth effect can change at more advanced stages of development.

The overall impact of inequality on growth is still a debated empirical issue. However the predictions here are consistent with most of the recent

empirical evidence. Work on the relationship between inequality and growth tends to be impaired by reverse causality problems and by the lack of reliable data for a sufficiently large number of countries. To reduce these problems, Easterly (2007) instruments for inequality using some physical characteristics of the countries' land. He finds an overall negative impact on growth. Still, there is evidence suggesting that this impact may be nonlinear and may even turn positive for the group of richer countries. In fact, Deininger and Squire (1998) found that inequality reduces income growth in poor countries but not in rich ones. Likewise Forbes (2000) found a positive relationship between inequality and growth using a sample that included mostly developed countries. This nonlinear effect between inequality and growth can be explained by the negative consequences of credit constraints on human capital (Galor and Zeira, 1993), as long as human capital is the relatively more scarce production factor in poor countries, and these constraints are loosened as countries become richer. However Barro (2000, 2008) still finds a nonlinear effect of inequality on growth after controlling for schooling as well as other possible mechanisms suggested by the literature (such as the investment ratio and rule of law). Specifically he finds a negative impact of inequality on growth among the group of less developed countries, which vanishes or even turns into a positive impact among the group of richer countries. The results in this working paper, just outlined, are entirely consistent with this evidence <sup>6</sup>.

The working paper is organized as follows. In the next section, we review some additional related literature on growth. The partial equilibrium analysis of the consumer's quantity/quality decisions is carried out in section 3. Section 4 embeds this analysis into an exogenous-technological-change

<sup>6.</sup> There is also some scattered historical evidence supporting a favorable contribution of lower income inequality to early industrializations, although its quantitative importance is unclear. More unequal economies would spend a larger share of income in artisan production of luxury goods consumed by the elite, which has a limited scope for productivity gains. In contrast, more equal distributions of income would be more favorable to industrial mass production and, therefore, to technical progress. For example, the United States developed a larger spectrum of massproduction industries than England (where production was often oriented towards higher quality) did in the first half of the nineteenth century. Apparently it was the consequence of a large demand by a wide number of middle class farmers and was at the root of its economic superiority in the twentieth century (Rosenberg, 1972). The data by Milanovic, Lindert and Williamson (2008) are also consistent with the first industrial countries having relatively low inequality extraction ratios at the time of industrialization (the inequality extraction ratio is a measure of actual inequality with respect to the maximum feasible inequality given the economy's resources). The model in this working paper captures in a simple way the positive relationship at early stages of development between more equality, mass production, and technical progress at producing low quality goods, as opposed to artisan production of high quality goods and low technical progress.

growth model. It explores the quantity/quality nature of GDP growth along different stages of development. Technological change is endogenized in section 5 assuming a two dimensional learning-by-doing process in a model where technological progress can be quality biased. This section also explores the potential role of progressive consumption taxes as a growth policy. Section 6 introduces a heterogeneity agent to study how income inequality affects growth through the quantity/quality composition mechanism. Section 7 summarizes and concludes.

### Related Growth Literature

f THIS working paper can be framed within the related growth literature as follows. Economies expand their per capita consumption as they get richer by consuming more units of each good (the intensive margin), higher average quality (the quality margin) and a wider set of goods (the extensive margin). Although most growth literature focuses on the intensive margin, the extensive margin is analyzed from different perspectives in Grossman and Helpman (1991) and Greenwood and Uysal (2005) among others. This working paper focuses on the joint evolution of the intensive and the quality margins taking the set of goods that individuals may choose to consume as constant. Grossman and Helpman (1991) also analyze quality-improving innovations across the existing set of consumption goods. However different qualities are assumed to be perfect substitutes. Hence all individuals consume the cheapest quality-adjusted good, and there is no room for a relationship between income and the demand for quality. The closest growth model with quality differentiated consumption goods is Stokey (1988). In Stokey (1988), as in this working paper, consumption expands in both the quantity and the quality dimensions, and a positive relationship between income and the quality of consumption is obtained. Nonetheless both the formulation and the applications here are different. A primary difference is that preferences in Stokey (1988) do not have enough structure to generate a specific pattern of the quantity/quality composition of growth. That is, output quality is increasing over time, but quality growth may or may not be increasing as a component of total growth. The time constraint here introduces such an structure. There is also an emphasis in this working paper to build a very tractable model that is easily amenable to quantitative analysis.

Demand nonhomotheticities create a link between income distribution and the composition of output, which in turn may affect efficiency, technical progress, and growth. There is a string of literature exploring these links to which this working paper is also closely related. Murphy, Shleifer and Vishny (1989) is a static model where lower inequality may help overcome the indivisibilities involved in modern industrial technologies. Zweimüller

(2000), Matsuyama (2002), and Foellmi and Zweimüller (2006) are closer to this working paper. They build increasingly richer dynamic models where income distribution affects efficiency and technical progress. However their setting is very different from the one in our working paper. All those works assume hierarchical preferences across a growing set of goods. Each individual consumes one or zero units of each good, and richer individuals consume all the goods being consumed by poorer individuals, plus some additional goods. Hence those studies focus on the extensive margin of consumption. Instead, the model in this working paper is set in a quantity/quality space of preferences and a two-dimensional space of technology allowing for different degrees of quality-biased technical progress. Within this framework, our working paper explores the intensive and quality margins of consumption across different stages of development, and the changing nature of technical progress in relation to these two dimensions of consumption. These differences bring about a distinctive mechanism and specific results about the interaction between inequality and growth <sup>7</sup>.

<sup>7.</sup> Two other related studies are the following: Mani (2001) investigates how the interaction between inequality and output composition affects human capital accumulation. Zweimüller and Brunner (2005) consider an economy with a quality-differentiated good and a nondifferentiated good in order to analyze how income distribution affects the intensity of innovative activities in the quality-differentiated good. Depending on the equilibrium regime, the three classes of consumers (poor, middle class, and rich) may or may not consume the same quality of the differentiated good; and lower inequality may have a positive effect of innovation. See Foellmi and Zweimüller (2006) for further references on related literature.

# 3. Time to Consume and the Preference for Quality

THIS section analyzes the basic microeconomics of quantity/quality consumption decisions in a partial equilibrium framework. The analysis takes prices and income as exogenous. In order to clarify the main assumptions in the analysis, it is useful to first consider an economy where there is only a single quality variety of the good.

#### 3.1. One good and a single quality

There is a single good which is produced in a single quality variety. Utility depends on the number of units x being consumed and the time allocated to their consumption. The more time that is allocated to consuming a given unit of a good, the more utility it provides. The time allocated to the consumption of each unit of consumption is the same for all units and denoted by  $\omega$ <sup>8</sup>. Consumer's utility function is

$$U = U(x, \omega),$$

$$\lim_{x \to 0, \omega \to \infty} U \ge 0; \ U_x > 0, \ U_\omega > 0,$$

$$\lim_{x \to 0} U_x - \frac{1}{x^2} U_\omega > 0; \ \frac{d}{dx} \left( U_x - \frac{1}{x^2} U_\omega \right) < 0,$$
(3.1)

where subscripts on U indicate partial derivatives. Total time allocated to consumption is exogenous and normalized to be 1  $^9$ . Normalize also the

<sup>8.</sup> This assumption could be derived from a framework where the consumer chooses how much time is allocated to the consumption of each unit of the good, and where the marginal utility of the time allocated to each unit of consumption decreases.

<sup>9.</sup> The complementarity between consumption and time has important implications for the allocation of time between work and leisure (or consumption). For example, in the case of

price of the single good to be 1 and denote income by y. Consumer maximizes (3.1) subject to income and time-for-consumption constraints:

$$x \le y, \tag{3.2a}$$

$$1 = x \cdot \omega, \tag{3.2b}$$

$$x \ge 0, \, \omega \ge 0. \tag{3.2c}$$

To interpret the assumptions on the utility function note that when the consumer increases the quantity of consumption, the amount of time available to consuming each unit of the good decreases by  $1/x^2$ . Therefore the overall increase in utility is  $U_x - U_\omega/x^2$ . Hence the assumptions  $\lim_{x\to 0} U_x - U_\omega/x^2 > 0$  and  $\frac{d}{dx} (U_x - U_\omega/x^2) < 0$  mean that, once we take into account the effect of increasing the quantity of consumption on the time to consume each unit, the overall marginal utility of the quantity of consumption is positive at x = 0 and decreasing x = 00. For every x = 00, assumptions on the utility function guarantee the existence of a unique optimal quantity of consumption  $x = x^*(y)$ , x = 00, involving per-unit of consumption time x = 01.

In the real word, consumers can enjoy many different goods at the same time. For example, one can simultaneously enjoy wearing a nice shirt, listening to music and having a drink. Still, there is a physical limit to the number of shirts, concerts, and drinks one can enjoy per unit of time. This motivates the following assumption that will provide the foundations for stronger results on the relationship between income and the demand for quality:

**Assumption 1.** There exists 
$$\overline{x}$$
 such that  $U_x\left(\overline{x}, \frac{1}{\overline{x}}\right) - \frac{1}{\overline{x}^2}U_\omega\left(\overline{x}, \frac{1}{\overline{x}}\right) = 0.$ 

That is, for a volume of consumption sufficiently large, the time allocated to consuming each unit of the good would be so small that the marginal utility of further increasing the quantity of consumption would be zero or even negative. Under this assumption, utility maximization

consumption and time being perfect complements and no quality differentiation, higher wage necessarily implies lower labor supply (then introducing quality differentiation opens the possibility of an upward sloping labor supply). However the main points in this working paper can be made without considering individuals' labor supply decisions.

10. A set of sufficient conditions for  $\frac{d}{dx}(U_x - \frac{1}{x^2}U_\omega) < 0$  is:  $U_\omega \ge 0$   $\rho_x = -xU_{xx}/U_x > 1$ ,  $\rho_\omega = -\omega U_{\omega\omega}/U_\omega > 1$ .

implies  $\lim_{y\to\infty} x^*(y) = \overline{x}$ . Thus as a consequence of the time constraint and the complementarity between consumption and time, individuals may look satiated even if the utility function does not display any satiation point  $^{11}$ .

#### 3.2. A continuum of qualities

Let us now introduce quality. There is a single good in the economy which can be produced along a continuum of quality varieties  $q \in [0, \infty)$ . Utility depends on the number of units x being consumed, their quality q, and the per-unit of consumption time  $\omega$ . The more time that is allocated to consuming a given unit of the good, the more utility it provides. Higher-quality varieties provide more utility than lower-quality varieties when allocating the same amount of time to their consumption. We simplify by considering separable utility functions of the form:

$$W(x, \omega, q) = U(x, \omega) \cdot V(q),$$

$$V(0) \ge 0; V_q > 0; \rho_q = -q V_{qq} / V_q > 0, \rho_q < \infty,$$

$$(3.3)$$

where  $U(x, \omega)$  is the same function (satisfying the same conditions) as in the previous subsection. As before, the time allocated to consumption is taken as exogenous and is normalized to be equal to 1. Individuals maximize (3.3) subject to

$$y = x \cdot p(q), \tag{3.4a}$$

$$1 = x \cdot \omega, \tag{3.4b}$$

$$x \ge 0, \, \omega \ge 0, \, q \ge 0, \tag{3.4c}$$

<sup>11.</sup> For example, it is unlikely that the number of shoes, cars, hotel rooms, etc. being consumed would go to infinity should their prices go to zero. In fact, it is hard to think of any good that would provide strictly positive marginal utility when the number of units being consumed per unit of time is unboundely increased (while maintaining constant their quality).

where p(q) is price as a function of quality. By an appropriate relabelling of quality varieties and without loss of generality, we can assume the following pattern of prices <sup>12</sup>:

$$p(q) = e^q. \tag{3.5}$$

Hence  $(\partial p/\partial q)/p = 1$ . Utility function (3.3) is assumed to be written after this relabelling.

Taking into account (3.5), utility maximization yields the following first order condition:

$$[xU_x - \omega U_\omega] / U = Vq / V. \tag{3.6}$$

Define u(x) = U(x, 1/x). The following assumption guarantees the second order conditions of utility maximization.

#### **Assumption 2.** $\rho_u = -xu_{xx}/u_x > 1^{-13}$ .

Expression (3.6) together with (3.4b) determine an increasing relationship between x and q, which is denoted  $q = \psi(x)^{14}$ . Taking derivatives in (3.6) and recalling assumption 2 yield:

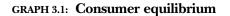
$$\psi' = \frac{q}{x} \frac{(\rho_u - 1) + V_q / V}{q V_q / V + \rho_q} > 0.$$
 (3.7)

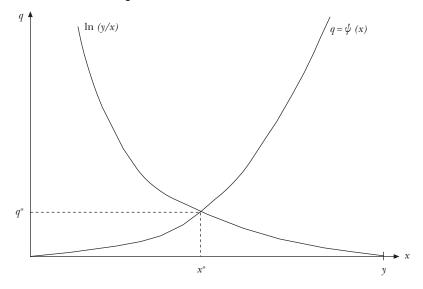
Given the individual's income y > 0, attainable pairs (x, q) are obtained by substituting with (3.5) in (3.4a):  $q = \ln (y/x)$ . Both schedules  $q = \psi(x)$  and  $q = \ln (y/x)$  are drawn in graph 3.1. Their intersection (together with [3.4b]) determines equilibrium values  $(x^*, \omega^*, q^*)$ . Clearly for any y > 0, we have  $x^* > 0$ ,  $\omega^* > 0$ ,  $q^* > 0$ . In turn, higher y involves higher  $x^*$  and  $q^*$ . Hence we have the following:

<sup>12.</sup> To see that this can be done without loss of generality, consider the original set of quality varieties to be  $Q \in [0, \infty)$ . Quality varieties are assumed to be labelled according to an increasing invariable preference order. Let P(Q) be the price of quality Q. P(Q):  $[0, \infty) \to [1, \infty)$  is assumed to be continuous, increasing, differentiable, and satisfying P(0) = 1. Relabel varieties as  $q \in [0, \infty)$  using the one-to-one increasing mapping  $\phi(Q) = q$  defined as  $\phi(Q) = \ln P(Q)$ . Hence  $Q = P^{-1}(e^q)$ . Therefore prices of each relabeled quality variety are given by  $p(q) = P(P^{-1}(e^q)) = e^q$ .

<sup>13.</sup> Again, a set of sufficient conditions for this assumption to hold is:  $U_{x\omega} \ge 0$ ,  $\rho_x = -xU_{xx}/U_x > 1$ ,  $\rho_\omega = -\omega U_{\omega\omega}/U_\omega > 1$ . Furthermore assumption 2 implies  $\frac{d}{dx}(U_x - \frac{1}{x^2}U_\omega) < 0$ .

<sup>14.</sup> If assumption 1 holds, this mapping is not defined over  $(0, \infty)$  but over  $(0, \overline{x})$ ; i.e.,  $\psi(x)$ :  $(0, \overline{x}) \to (0, \infty)$ .





**Proposition 3.1.** Let assumption 2 hold. As income rises, the quality of consumption increases.

The empirical evidence cited in the Introduction suggests that quality growth is an increasingly important component of GDP growth. For this to happen, it is not enough that quality is increasing in income. It is also necessary that as income rises, an increasing share of every income rise is spent upgrading the quality of consumption. Next, we show that assumption 1 implies such behavior. From (3.4a), the share of an income rise dy, that is spent and so increasing the quantity of consumption, is  $p(q) \frac{dx}{dy} = \frac{1}{1 + x\psi'}$ . Symmetrically the share of an income rise, that is spent upgrading the quality of consumption, is

$$x \frac{\partial p}{\partial q} \frac{dq}{dy} = \frac{x \psi'}{1 + x \psi'}.$$
 (3.8)

The following proposition conveys the message that as income rises, the share of every income rise, that is spent upgrading the quality of consumption, is increasingly larger.

**Proposition 3.2.** Let assumptions 1 and 2 hold. For any s, 0 < s < 1, there is an income level y (s) sufficiently large such that for y > y (s), a share larger than s of every income rise is spent upgrading the quality of consumption.

**Proof.** Assumption 1 implies that  $\lim_{y\to\infty} x(y) \le \overline{x}$ . Hence from the budget constraint  $y = xe^q$ , we have  $\lim_{y\to\infty} q = \infty$ . Recall that  $\rho_q$  is bounded from above, and note that since V is concave,  $qV_q/V$  is bounded below 1. Therefore  $\frac{(\rho_u-1)+V_q/V}{qV_q/V+\rho_q}$  in (3.7) is bounded from below above zero. Hence  $\lim_{y\to\infty} x\psi' = \infty$ , implying  $\lim_{y\to\infty} x\frac{\partial p}{\partial q}\frac{dq}{dy} = \frac{x\psi'}{1+x\psi'} = 1$ . Therefore since (3.7) is continuous, for any s, 0 < s < 1, there is an income level y (s) sufficiently large such that y>y (s) implies  $x\frac{\partial p}{\partial q}\frac{dq}{dy}>s$ .

# 4. The Quantity/Quality Composition of Growth

In this section, we lay out an exogenous growth model to explore the quantity/quality composition of gross domestic product (GDP) growth. There is a single representative agent and a single good that can be produced along a continuum of quality varieties  $q \in [0, \infty)$ .

#### 4.1. Technology

Labor is the only factor of production, and there are constant returns to scale to produce any quality variety. Producing higher quality requires more labor per unit of output, which is given by a function  $F(q): [0, \infty) \to [1, \infty)$ . F(q) is continuous, differentiable, and strictly increasing; it also satisfies F(0) = 1. Thus output at time t,  $x_t$ , when producing quality  $q_t$ , is given by

$$x_t = \frac{A_t L}{F(q_t)} ,$$

where L is the labor input, and  $A_t$  is a general efficiency parameter that evolves over time. The labor supply is assumed to be constant and is normalized as L=1. By an appropriate relabelling of quality varieties, we can assume  $F(q)=e^{q/\gamma}$  without loss of generality <sup>15</sup>. Therefore the production function using the relabelling for the quality varieties is given by:

$$x_t = \frac{A_t}{e^{\eta t/\gamma}}. (4.1)$$

15. To see this, consider the original set of quality varieties to be  $Q \in [0, \infty)$ . Unit labor requirements for production are given by F(Q); F(Q):  $[0, \infty) \to [1, \infty)$ , F(0) = 1, F' > 0. Relabel varieties as  $q \in [0, \infty)$  using the one-to-one increasing mapping  $\psi$  (Q) = q defined as  $\psi$  (Q) =  $\gamma$  ln  $F(Q_t)$ . Hence  $Q = F^{-1}(e^q/\gamma)$ . Therefore substituting in the production function  $x_t = A_t L_t/F(Q_t)$  yields:  $x_t = A_t L_t/e^{qt/\gamma}$ . Note that this is a commonly used production function in models with quality differentiation (see, for example, Flam and Helpman, 1987).

Denote the wage at t by  $w_t$ . Assuming perfectly competitive markets, prices are given by

$$p_t(q) = \frac{e^{q/\gamma}}{A_t} \omega_t. \tag{4.2}$$

Therefore  $(\partial p_t/\partial q_t)/p_t = 1/\gamma$ . In this section, technological progress is exogenous:

$$g(A_t) = \theta > 0, \tag{4.3}$$

where g(.) indicates growth rate of the variable in parenthesis.

#### 4.2. The composition of GDP growth

Consider the same utility function and constraints as in the previous section leading to the increasing relationship  $q_t = \psi(x_t)$ . Given  $A_t$  expression (4.1) determines the feasible pairs  $(x_t, q_t)$  whereas  $q_t = \psi(x_t)$  determines consumers' choice among those pairs. The intersection between (4.1) and  $q_t = \psi(x_t)$  determines the instantaneous equilibrium pair  $(x_t, q_t)$  at time t. See graph 4.1. Clearly for any  $A_t > 0$ , there is a unique pair  $(x_t, q_t) > (0, 0)$  solving this system. As  $A_t$  grows,  $x_t$  and  $q_t$  will keep increasing.

Denote GDP at time t by  $y_t$ :

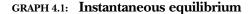
$$y_t = x_t p_t (q_t).$$

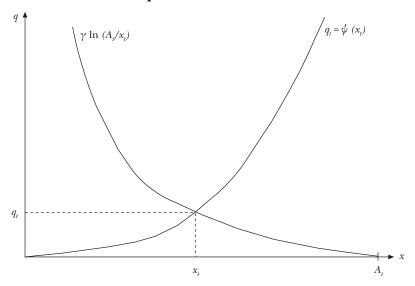
Therefore GDP growth at the constant prices for each quality variety  $p_t(q)$  is given by  $g(y_t) = g(x_t) + \frac{\partial p_t(q)/\partial q_t}{p_t(q)} \frac{q_t.\text{Since }(\partial p_t/\partial q)/p_t = 1/\gamma, \text{ and since we can also obtain } g(y_t) \text{ from } y_t = \frac{A_t}{e^{tt/\gamma}} p_t(q_t), \text{ it yields:}$ 

$$g(y_t) = g(x_t) + \frac{q_t}{\gamma} g(q_t) = g(A_t) = \theta.$$
 (4.4)

This expression divides GDP growth into a quantity and a quality component. From (3.8) and (4.4), the quantity component of GDP growth is given by:

$$g(x_t) = \frac{1}{1 + x_t \psi'} g(y_t). \tag{4.5}$$





**Proposition 4.1.** Let assumptions 1 and 2 hold. For any s, 0 < s < 1, there is a level A(s) of development in terms of technological efficiency such that for A > A(s), the share of quality growth in total GDP growth is larger than  $s(i.e., \frac{q_t}{\gamma} g(q_t) > s \cdot g(y_t))$ .

**Proof.** This proposition is an immediate implication of the constant positive growth  $g(A_t) = \theta$ , expression (4.4), and proposition 3.2. constant positive growth  $g(A_t) = \theta$  and  $g(y_t) = g(A_t)$  yield  $\lim_{t \to \infty} y_t = \infty$ . Then, proposition 3.2 implies  $\lim_{t \to \infty} x_t \psi' = \infty$ . Therefore (4.5) yields  $\lim_{t \to \infty} g_x(t) = 0$ . Hence eventually, quantitative growth plays no role in the composition of GDP growth. Symmetrically  $\lim_{t \to \infty} \frac{q_t}{\gamma} g(q_t) = \lim_{t \to \infty} g_y(t) = \theta$ . Since all the variables are continuous as a function of time (or as a function of  $A_t$ ), the result in the proposition follows.

### 5. Endogenous Growth: Quality-Biased Technical Progress

THIS section builds an endogenous growth model, where technical progress is due to learning by doing. Repetition of tasks improves skills and knowledge that help both the speed and quality of work. Hence technical progress comes along two dimensions:

- I) general efficiency in producing any quality variety, and
- II) relative efficiency in producing higher qualities. Economies producing output of higher quality give rise to learning that is more quality biased <sup>16</sup>.

#### 5.1. The model

**Technology.** Consider the same production function as before except that now both technological parameters  $A_t$  and  $\gamma_t$  are subject to progress due to learning-by-doing. Labor supply is constant and normalized to be 1. Thus output at time t, when producing quality  $q_t$ , is given by:

$$x_t = \frac{A_t}{e^{q_t/\gamma_t}}. (5.1)$$

As before,  $A_t$  is a general efficiency parameter, whereas  $\gamma_t$  governs the relative efficiency in producing higher-quality goods. Both parameters  $A_t$  and  $\gamma_t$  evolve over time as a result of learning by doing. In the case of gen-

<sup>16.</sup> Learning by doing may be considered as a first simple formulation for technical progress. Still it seems likely that a model with intentional R+D would bring about the same results: technical progress is increasingly quality-biased. The reason is that the incentives for quality-biased R+D (as oposed to quality neutral R+D) are likely to rise, as increases in demand become circumscribed to the quality dimension.

eral efficiency, we consider a learning-by-doing process similar to the one in Krugman (1987) and Lucas (1988):

$$A_t = \theta_1 \ x_t - \delta_1 \ A_t; \quad \theta_1 > \delta_1 > 0.$$
 (5.2)

In turn, learning affecting  $\gamma_t$  is linked to both the quantity and quality being produced. For example, accumulated experience in sewing shirts in a clothing factory may bring about not only a higher number of sewed shirts per unit of labor but improvements in skills and techniques to carry out more accurate seams. Or, in package delivery, accumulated experience may help develop techniques to reduce misplacements (which is a quality characteristic), besides increasing the number of deliveries per worker. In general, repeating a task helps improve not only the speed at which it is performed but also the quality of the result. Moreover the higher the output quality being targeted in a production process, the more likely it is that generated learning will be quality-biased. Still as the number of units being produced tends to zero, learning will also tend to zero even if the quality being produced is very high. These circumstances are embedded in the following learning process:

$$\gamma_t = \theta_2 \ q_t \ x_t - \delta_2 \ \gamma_t; \qquad \theta_2 > \delta_2 \ge 0. \tag{5.3}$$

Obsolescence parameters  $\delta_1$  and  $\delta_2$  may be justified in terms of a succession of finitely lived representative agents, whose skills have to be replaced <sup>17</sup>. At any rate, results are obtained assuming that  $\delta_1$  and  $\delta_2$  are small. In fact, the same basic results can be obtained with  $\delta_1 = \delta_2 = 0$ . However  $\delta_1 > 0$  brings about the existence of a steady state with constant rates of growth that is particularly easy to work with.

**Utility.** We now simplify by considering a particular case of the utility function in previous sections, which can deliver a closed form solution. The representative consumer maximizes the following instantaneous utility function subject to the same constraints (3.4a)-(3.4c) as before:

$$U_t = \frac{\omega_t}{e^{\sigma/x_t}} q_t. \tag{5.4}$$

As in the simpler model of the previous section, expression (5.1) implies that perfect-competition prices are given by:

<sup>17.</sup> Knowledge, experience, and skills are embodied in individuals that are subjet to a life cycle.

$$p_t(q) = \frac{e^{q/\gamma_t}}{A_t} \omega_t. \tag{5.5}$$

Therefore,  $(\partial p_t/\partial q)/p_t = 1/\gamma_t$ . This together with the first order conditions of utility maximization bring about the following relationship between quantity and quality:

$$\frac{q_t}{\gamma_t} = \frac{x_t}{\sigma - x_t} \,. \tag{5.6}$$

#### 5.2. Equilibrium and growth

**Instantaneous equilibrium.** Define  $z_t = q_t/\gamma_t$ . Expressions (5.1) and (5.6) can then be rewritten as:

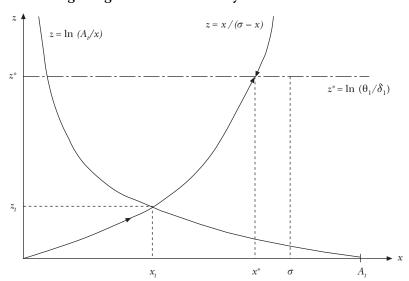
$$x_t = \frac{A_t}{e^{z_t}}. (5.7)$$

$$z_t = \frac{x_t}{\sigma - x_t}. ag{5.8}$$

Given  $A_t$ , the intersection of (5.7) and (5.8) determines the instantaneous equilibrium pair  $(x_t, z_t)$  at time t. See Graph 5.1. In particular, (5.7) determines the feasible pairs, whereas (5.8) determines consumers' choice among those pairs. Note that  $x/(\sigma-x_t)$  is increasing in x>0 with  $\lim_{x\to 0} \frac{x_t}{\sigma-x_t} = 0$ , whereas  $\ln(A_t/x)$  is decreasing in x. Clearly for any  $A_t>0$ , there is a unique pair  $(x_t, z_t)>(0,0)$  solving (5.7)-(5.8). Both  $x_t$  and  $z_t$  are continuous and strictly increasing in  $A_t$ . Then given  $\gamma_t$ ,  $q_t=z_t\gamma_t$  determines output quality  $q_t$ .

**Gross domestic product (GDP) growth.** As in the previous section, denote GDP, at time t,  $y_b$  is given by  $y_t = x_t p_t (q_t)$ . GDP growth, at time t at constant prices  $p_t(q)$ , is given by  $g(y_t) = g(x_t) + \frac{\partial p_t(q)/\partial q_t}{p_t(q)} q_t$ . Since  $(\partial p_t/\partial q)/p_t = 1/\gamma_t$  and using (5.1), we have

$$g(y_t) = g(x_t) + \frac{q_t}{\gamma_t} g(q_t) = g(A_t) + \frac{q_t}{\gamma_t} g(\gamma_t).$$
 (5.9)



GRAPH 5.1: Long run growth and the steady state

This expression provides two approaches to GDP growth. From the point of view of the composition of output, GDP grows in both the quantity and the quality dimensions. From the point of view of its source, GDP grows due to general, as well as quality-biased, technical progress. Note that the *GDP-growth value* of raising the quality of consumption  $g(q_t)$  and of improving efficiency in producing higher quality  $g(\gamma_t)$  depends on the elasticity of prices with respect to quality:  $q_t/\gamma_t = (\partial p_t/\partial q) q_t/p_t = z_t$ . Also note that there is no reason to expect  $g(x_t)$  to be equal to  $g(A_t)$ , and  $g(q_t)$  to be equal to  $g(\gamma_t)$  (in fact, this is the case only in the steady state). The remaining of this section focuses on the analysis of GDP growth from the point of view of its technical progress source.

From (5.7), (5.2) and (5.3) we have:

$$g(A_t) = \frac{\theta_1}{e^{z_t}} - \delta_1. \tag{5.10}$$

$$g(\gamma_t) = \theta_2 z_t x_t - \delta_2. \tag{5.11}$$

Using these two equations to substitute in (5.9), it yields the following expression for GDP dynamics:

$$g(y_t) = \frac{\theta_1}{e^{z_t}} + \theta_2(z_t)^2 x_t - (\delta_1 + \delta_2).$$
 (5.12)

Note from (5.11) that, given the parameters of the economy and initial technological conditions, nonnegative growth rates of  $\gamma_t$  (and of GDP) require  $\delta_2$  to be sufficiently low.

**Steady state.** This economy has a unique steady state with strictly positive growth, where the volume of output is constant whereas quality grows at a constant rate. The following proposition characterizes it using asterisks to denote steady state values.

**Proposition 5.1.** For  $\delta_2$  sufficiently small, the model has a unique steady state with  $A^* > 0$ ,  $x^* > 0$ ,  $z^* > 0$ ,  $g_q^* = g_\gamma^* > 0$ , and  $g_\gamma^* = z^* g_\gamma^*$ . Moreover  $g_\gamma^*$  is increasing in  $\theta_1$ ,  $\theta_2$ , and  $\sigma$ , and decreasing in  $\delta_1$  and  $\delta_2$ .

#### **Proof.** See appendix.

In graph 5.1, the  $z = \ln (A_t/x)$  schedule shifts over time towards the North-East until it crosses the  $z_t = x_t/(\sigma - x_t)$  schedule at point  $z^* = \ln (\theta_1/\delta_1)$ , which corresponds to the steady state.

**Transitional dynamics.** The system (5.7)-(5.8)-(5.10) can be solved independently from the rest of the equations. Denote the initial condition by  $A_0$ . For  $A_0 < A^*$ , the schedule (5.7) at t = 0 crosses (5.8) to the South-West of  $(x^*, z^*)$  in graph 5.1. It is easy to see that the economy converges monotonically to its steady state <sup>18</sup>.

Now we characterize GDP growth from the point of view of the quality bias of the technical progress involved. Using expression (5.9), in particular  $g(y_t) = g(A_t) + z_t g(\gamma_t)$ , we can analyze the relative importance of quality-neutral technical progress  $g(A_t)$  and quality-biased technical progress  $z_t g(\gamma_t)$  in generating growth at different stages. Recall that early stages of development are characterized by a level of general efficiency A close to zero.

**Proposition 5.2.** Technical progress is mostly quality-neutral at early stages of development but becomes increasingly quality-biased over time.

<sup>18.</sup> The formal argument is as follows. Given  $0 < A_0 < A^*$ , (5.7) - (5.8) imply  $0 < x_o < x^* < \sigma$ ,  $0 < z_0 < z^* = \ln (\theta_1/\delta_1)$ . Then,  $g(x_t) = g(A_t) \frac{\sigma - x_t}{2\sigma}$  (which can be obtained using [5.10], [5.7] and [5.8]) implies  $g(A_t) > 0$  and  $g(x_t) > 0$ . Then,  $g(z_t) = g(x_t)[1 + z_t]$  implies  $g(z_t) > 0$ . Therefore the system converges monotonically to  $(A^*, x^*, z^*)$ .

**Proof.** First consider the instantaneous equilibrium at initial stages of development which are characterized by A(t) close to zero. Note that  $\lim_{A_t \to 0} g(A_t) = \theta_1 - \delta_1 > 0$ ,  $\lim_{A_t \to 0} z_t = 0$ ,  $\lim_{A_t \to 0} x_t = 0$ ,  $\lim_{A_t \to 0} g(\gamma_t) = \lim_{A_t \to 0} \theta_2 z_t x_t - \delta_2 \le 0$ , and that growth rates are continuous in time. Therefore for A(t) sufficiently close to zero, we have  $g(A_t) > g(\gamma_t) > z_t g(\gamma_t)$ . Therefore from expression (5.12), we have that at early stages of development, quality-neutral technological progress  $g_A$  is the main source of growth.

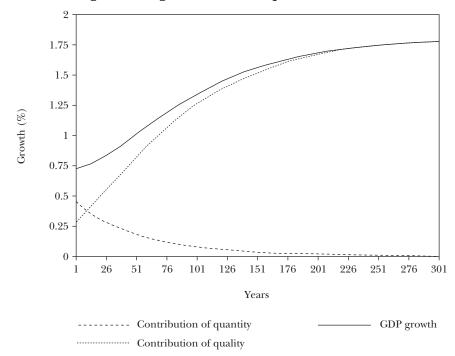
Now consider later stages of development. Recall that  $z_t$  increases monotonically to its steady state. Hence expression (5.10) implies that  $g(A_t)$  decreases monotonically from  $g_A(0) > 0$  to  $g_A^* = 0$ . On the other hand, since  $z_t$  and  $z_t$  increase monotonically towards their steady state values, (5.11) implies that  $z_t g(\gamma_t)$  increases monotonically towards  $z_t^* g_{\gamma_t}^*$  which is strictly positive.

#### 5.3. A numerical illustration

This subsection shows that this framework may deliver reasonable long run paths for per capita GDP growth <sup>19</sup>. Neoclassical models predict decreasing growth rates, whereas standard endogenous growth models tend to predict constant rates. However the secular pattern of growth in advanced economies since the industrial revolution is one of increasing growth rates that stabilized somewhat below 2% in the last half century. For example, estimates of output per worker growth in England at the beginning of the nineteenth century go from 0.35% to 1.3% (see Feinstein, 1981 and Crafts and Harley, 1992). On the other hand, U.S. per capita growth in the last decades averages about 1.7% (though, as discussed in the introduction, this number seems to underestimate GDP growth due to nonmeasured quality growth). In this subsection, we show the results of discretizing and calibrating the model, so that it roughly matches the long run per capita growth path of the most advanced economies in the last two centuries.

The calibration generates an initial per capita GDP growth of 0.72% (which is roughly consistent with the mean of the cited estimates for England at the beginning of the xixth century) and yields a steady state growth of 1.8% (which is consistent with the overall observed trends in the last decades). Parameter values are set as follows. Since skills and knowledge are embodied in individuals, annual depreciation rates  $\delta_1 = \delta_1 = 0.025$  seem

<sup>19.</sup> Although the stripped-down simplicity of the technical progress functions prevents obtaining plausible decompositions between quantity and quality growth.



GRAPH 5.2: Long run GDP growth and its composition. A numerical illustration

reasonable. Furthermore we set  $\sigma = 3$ ,  $\theta_1 = 0.054$ ,  $\theta_2 = 0.048$ . Finally the initial value  $A_0$  is chosen such that  $x_0 = 1$ . Graph 5.2 draws the annual rates of growth of GDP for the first 300 years. As noted, calibrated GDP growth starts at an annual rate of 0.72% and then shows a rapid acceleration, doubling after 120 years and surpassing 1.68% after 150 years.

GDP growth is also decomposed in graph 5.2 into the quantity growth component,  $g(x_t)$ , and the quality growth component,  $z_t g(x_t)$ . Consistent with proposition 5.1, quantitative growth is the main source of growth at initial stages. Then it is gradually substituted by quality growth. Although an empirical assessment of the secular paths of these two components of growth does not yet exist, their paths in graph 5.2 seem implausible (the calibration seems to overrate quality growth in late periods). Future work may find necessary to introduce more flexible functional forms for technical progress in order to generate more reasonable paths of these two components as estimates of their secular paths become available  $^{20}$ .

<sup>20.</sup> Note that considering population growth would only add to quantity GDP growth and would likely reduce the quality bias of technical progress. Thus introducing population growth in the model would also likely reduce the relative importance of the quality component of growth.

#### 5.4. Growth policy: consumption taxes

Progressive consumption taxation (i.e., taxing luxury goods at higher rates) has been discussed from different perspectives such as its impact on aggregate savings and the distribution of wealth <sup>21</sup>. In this section, we discuss its possible role as a growth policy. Note that different quantity/quality compositions of output give rise to different technical progress externalities in terms of their intensity and quality bias. Moreover the *value* of quality-biased technical progress changes along different stages of development: as individuals consume higher quality, quality-biased technical progress becomes more valuable for growth. In this context, distortionary consumption taxes (or subsidies) can be used to influence the quantity/quality composition of output in order to favor the most valuable combination of externalities at each point of time.

First, let us see how the government can influence the pair  $(x_t, z_t)$  within the feasible pairs given by (5.7) by using a nonlinear tax/subsidy scheme. Denote after-tax/subsidy prices by  $p_{\tau}(q, t)$ . Consider the following nonlinear tax/subsidy scheme parameterized by  $\tau > 0$ :  $p_{\tau}(q, t) = [p(q, t)]^{\tau}$ . Note that  $\tau > 1$  (respectively,  $\tau < 1$ ) involves a tax/subsidy scheme relatively unfavorable (respectively, favorable) to the higher-quality goods. Consumers are assumed to finance these subsidies or receive the yields of this tax scheme in a lump sum. The production function in (5.7), together with perfect competition, yields:

$$p_{\tau}(q, t) = \eta \left[ \frac{-\ell^{\eta_t/\gamma_t}}{A_t} \omega_t \right]^{\tau}.$$

Plugging these new prices into the income constraint and maximizing utility yield a new version for expression (5.8):

$$z_t \tau = \frac{x_t}{\sigma - x_t} \ . \tag{5.13}$$

This expression, together with (5.7), determines the new instantaneous equilibrium. Now taking derivatives in (5.13) yields:

$$\frac{dz_t}{d\tau} = \frac{x_t \left[\sigma - x_t\right]}{\sigma \left[\partial x_t / \partial z_t\right] - \tau \left[\sigma - x_t\right]^2} < 0,$$
(5.14)

<sup>21.</sup> See Frank (2005) and references therein.

where from (5.7) we have  $\partial x_t/\partial z_t = -x_t$ . Therefore, as expected, tax/subsidy schemes relatively unfavorable to higher quality goods reduce the quality of consumption and increase quantity. This can be used to affect growth: at stages where the most effective learning is linked to reaching a larger scale of output, reducing the demand for quality will enhance GDP growth. Promoting higher quality production may be optimal at other stages of development.

Consider the impact of a tax/subsidy scheme  $\tau$  on GDP growth. From (5.9), we have:

$$\frac{dg(y_t)}{d\tau} = \left[ \frac{dg(A_t)}{dz_t} + z_t \frac{dg(\gamma_t)}{dz_t} + g(\gamma_t) \right] \frac{dz_t}{d\tau} . \tag{5.15}$$

Changing au affects GDP growth through three components: the growth of  $A_b$  the growth of  $\gamma_b$  and the GDP value  $z_t$  of the growth of  $\gamma_t^{22}$ . As already shown, higher  $\tau$  increases the scale of production and reduces its quality. This is always positive for  $g(A_t)$  but may have an uncertain effect on  $g(\gamma_t)$ , and has a negative effect on  $z_t$ . The relative importance of these effects changes over time. Early stages of development are characterized by a low general efficiency  $A_{t}$  resulting in an output consumption  $x_{t}$  close to zero and minimal quality. Increasing the scale of output at these stages brings about a sizable increase in general efficiency growth A; whereas the alternative of increasing quality would not bring about much quality-biased technical progress because this also requires a large scale of output (which is hurt by increasing quality). Moreover quality-biased technical progress is of little value because individuals are consuming very low quality at these stages (hence  $z_t$  is very low). Therefore a tax/subsidy scheme that shifts GDP towards larger output, even if this involves lower quality, has a positive net effect on GDP growth. However the positive effect of a progressive tax scheme on  $g(A_t)$  becomes weaker at later stages of development as  $A_t$  increases. Meanwhile the negative effect on  $g(\gamma_t)$  becomes larger, while the value of being more efficient in producing higher quality also increases. Therefore for some parameter values, a tax/subsidy scheme that shifts consumption towards higher quality and lower quantity may have a positive effect on GDP growth at more advanced stages of development.

<sup>22.</sup> Note that z is the elasticity of the price with respect to quality:  $z = q/\gamma = (\partial p/\partial q)/(q/p)$ . If individuals consume the lowest possible quality (i.e.,  $q_i = 0$  implying  $z_i = 0$ ) increasing efficiency in producing higher quality has no impact on GDP growth.

**Proposition 5.3.** At early stages of development, progressive consumption taxes enhance GDP growth.

**Proof.** Consider the initial stages of development which are characterized by  $A_t$  being close to 0. This implies  $x_t$  and  $z_t$  close to 0 ( $\lim_{A_t \to 0} x_t = \lim_{A_t \to 0} z_t = 0$ ). Note from (5.10) that  $\lim_{A_t \to 0} dg (A_t)/dz_t = -\theta_1/e^{zt} < 0$ , whereas from (5.11) we have  $\lim_{A_t \to 0} dg (\gamma_t)/dz_t = 0$  and  $\lim_{A_t \to 0} g (\gamma_t) \le 0$ . Since  $dz_t/d\tau < 0$  (see [5.14]), we then have  $\lim_{A_t \to 0} dg (y_t)/d\tau > 0$ . Now continuity of these derivatives implies that at least for an initial interval of time where  $A_t$  is still sufficiently low, GDP growth is increasing in the tax scheme  $\tau$ .

This proposition will hold for a more general utility setting as long as  $dz_t/d\tau < 0^{23}$ . The proposition suggests that industrialization at early stages is favored by helping the scale of production at the cost of lowering quality. However once output has reached a large scale, growth may be more effectively enhanced by favoring the quality dimension. In fact, for some parameters, a subsidy to the production of higher quality goods,  $\tau < 1$ , will enhance growth. This will be the case, for example, for  $\delta_1$  and  $\delta_2$  close to zero and  $\ln (\theta_1/\delta_1) < 2^{24}$ .

$$=\left[-\frac{\theta_1}{e^{z_l}}+\theta_2\sigma\,\frac{z_l^2}{1+z_l}(2-z_l)-\delta_2\right]\frac{dz_l}{d\tau} \text{ and substituting with } z^*=\ln\ (\theta_1/\delta_1).$$

<sup>23.</sup> The reason is that the other elements in the proof are not the consequence of preferences but of technology. Consider, for example, utility functions delivering, from the first order condition, a relationship  $z_t \tau = \varphi(x_t)$ , for some increasing function  $\varphi(\cdot)$ . This is the case if we use utility in expression (3.3) and assumption 2. This implies  $\frac{dz_t}{d\tau} = \frac{z_t}{\varphi(x_t)'(dx_t/dz_t) - \tau} < 0$ .

<sup>24.</sup> In that case,  $z^* < 2$ . The argument is completed using (5.15) to get  $dg(y_t)/d\tau =$ 

6. Inequality and Growth:
The Quantity/Quality
Channel

THIS section analyzes the effects of income inequality on growth that are channelled through the quantity/quality composition of output. The mechanism is that inequality affects the quantity/quality composition of output which in turn influences the intensity as well as the quality bias of technical progress, and this affects growth. Higher inequality involves a higher average quality of output but smaller scale. Since a larger scale tends to be more beneficial to growth than higher average quality at early stages of development, higher inequality hinders growth at these stages. Nonetheless the sign of this effect can change at later stages: when the scale of production is already large and when consumers prefer upgrading the quality of consumption rather than augmenting its quantity, quality-biased technical progress—which can be favored by higher inequality—becomes specially effective in supporting gross domestic product (GDP) growth.

Inequality issues can be considered in the model by introducing a new productive asset: *land*. All individuals are assumed to supply the same amount of homogeneous labor per period, but land ownership is unequally distributed. The effects of inequality on growth are analyzed by considering exogenous changes in the distribution of land ownership.

**Technology.** Production of any quality variety uses labor as well as land according to the following Cobb-Douglas production function:

$$x_{q_t} = \frac{A_t}{e^{q_t/\gamma_t}} (N_{qt})^{\alpha} (L_{qt})^{1-\alpha}, \quad 0 < \alpha < 1,$$
 (6.1)

where  $x_{qt}$  is the number of units of quality q being produced at time t, and  $L_{qt}$  and  $N_{qt}$  are the amount of labor and land used in this production, respectively. Normalize the total supply of labor and land such that L = N = 1. Note that it is

optimal to produce all quality varieties with the same labor/land ratio. Hence,  $\left(\frac{N_{qt}}{L_{ot}}\right)^{\alpha} = \left(\frac{N}{L}\right)^{\alpha} = 1$ . Therefore,

corec, 
$$\left(\frac{L_{qt}}{L_{qt}}\right) = \left(\frac{L}{L}\right) = 1$$
. Therefore, 
$$x_{qt} = \frac{A_t}{\rho q_t \gamma_t} L_{qt}.$$

Define  $s_t$  as  $s_t = \omega_t + r_b$  where  $\omega_t$  is the wage and  $r_t$  is the rent obtained by a unit of land at time t. Perfect competition prices are:

$$p_{t}(q) = \omega_{t} \frac{L_{qt}}{x_{qt}} + r_{t} \frac{N_{qt}}{x_{qt}} = \frac{L_{qt}}{x_{qt}} \left[ \omega_{t} + \frac{N}{L} r_{t} \right] = \frac{e^{q/\gamma_{t}}}{A_{t}} s_{t}.$$
 (6.2)

Therefore GDP at time t,  $Y_t$ , is given by:

$$Y_t = \sum_{q} x_{qt} p_t (q) = s_t. \tag{6.3}$$

In turn, expressions (5.10) and (5.11) for the dynamics of technology become:

$$g_{At} = \frac{\theta_1}{A_t} \sum_{i} x_{it} - \delta_1. \tag{6.4}$$

$$g_{\gamma t} = \frac{\theta_2}{\gamma_t} \sum_i x_{it} \ q_{it} - \delta_2. \tag{6.5}$$

**Individuals' income.** Individuals are indexed by i (i = 1, ..., I), and all have the same utility function (5.4) assumed in the previous section. We adopt the strong simplification that in every period each agent's consumption is equal to his income. As already noted, income inequality is due to the unequal distribution of land ownership. Let  $\rho_i$  ( $0 \le \rho_i \le 1$ ) be individual i's share in the ownership of the total supply of land. Denote individual i's income by  $y_{ib}$  and his share in aggregate income by  $\beta_i$ :  $y_{it} = \beta_i Y_t$ . Since  $\alpha$  is the share of land rents in national income, and  $1 - \alpha$  is the share of wages, we have  $\beta_i = \rho_i \alpha + (1 - \alpha)/I$ .

Define  $x_{it}$  as the volume of agent i's consumption at time t,  $q_{it}$  its quality, and  $z_{it} = q_{it}/\gamma_t$ . Given  $A_t$  and expression (6.2), attainable pairs to individual i at time t,  $(x_{it}, z_{it})$ , satisfy  $\beta_i Y_t = x_{it} p_t (q_{it}) = x_{it} \frac{e^{z_{it}}}{A_t} s_t$ . Thus since  $s_t = Y_t$  individual i's attainable pairs at time t satisfy  $^{25}$ :

<sup>25.</sup> To be precise, expression (6.6) defines the *frontier of the set of* attainable pairs. The consumer always chooses points in this frontier, so that we can ignore the rest of the set. Note that since all

$$x_{it} = \frac{A_t}{e^{z_{it}}} \beta_i. \tag{6.6}$$

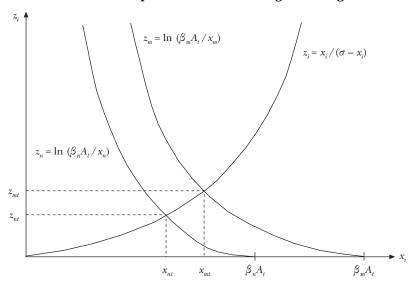
Instantaneous equilibrium and growth. Intersection of (6.6), with expression (5.8) from utility maximization, determines individual i's instantaneous equilibrium at time t (which, in turn, given  $\gamma_b$  determines optimal  $q_{it}$ ). See graph 5.3 where (6.6) is drawn for two individuals m and n such that  $\beta_m > \beta_n$ . Clearly,  $\beta_m > \beta_n$  implies  $(x_{mb}, z_{mt}) > (x_{nb}, z_{nt})$ .

Using (6.6) to substitute in (6.3) yields  $Y_t = \sum_i \frac{A_t}{e^{qi_t/\gamma_t}} \beta_i p_t(q_i)$ . Hence GDP growth at constant prices is given by:

$$g(Y_t) = g(A_t) + g(\gamma_t) \sum_i \beta_i z_{it}.$$
(6.7)

**Inequality and growth.** The impact on growth of changes in the distribution of income is analyzed by considering exogenous changes in the  $\beta_i$ s such that  $\sum_i \beta_i = 1$ . These changes implicitly involve changes in the distribution of land shares  $\rho_i$ . The inequality increases considered in the analysis are any asset redistribution from individuals with lower  $\beta_i$  to individuals with higher  $\beta_i$ .

#### GRAPH 5.3: Instantaneous equilibrium with heterogeneous agents



goods are produced with the same combination of land and labor,  $\beta_i$  may be seen as the share of the aggregate supply of these two combined inputs that individual i is able to buy in order to produce his consumption bundle.

Thus the effect of higher inequality on growth can be analyzed by investigating  $\frac{dg\left(Y_{t}\right)}{d\beta_{m}} - \frac{dg\left(Y_{t}\right)}{d\beta_{n}}$ , for  $\beta_{m} > \beta_{n}$ ; or by investigating  $\frac{d^{2}g\left(Y_{t}\right)}{d\beta_{n}^{2}}$ ,

where a negative value would imply a negative impact of inequality on growth.

From (6.7) the derivative of GDP growth with respect to  $\beta_i$  is given by

$$\frac{dg(Y_t)}{d\beta_i} = \frac{dg(A_t)}{d\beta_i} + \frac{dg(\gamma_t)}{d\beta_i} \sum_i \beta_i z_{it} + g(\gamma_t) \frac{d\sum_i \beta_i z_{it}}{d\beta_i}.$$
(6.8)

In expression (6.8),  $g(A_t)$  is increasing and concave in  $\beta_i$  since  $g(A_t)$  only depends on consumption quantities  $\sum_i x_{it}$ , and quantities are concave in income. Thus increasing inequality is negative for  $g(A_t)$ . In turn,  $g(\gamma_t)$  (which depends on  $\sum_i q_{it} x_{it}$ ) is increasing in  $\beta_i$  and initially convex, but can change to concave before the steady state is reached. Similarly,  $\sum_i \beta_i z_{it}$  is also convex in  $\beta_i$  for low  $z_{it}$ . Hence the sign of the relationship between inequality and growth through the quantity/quality mechanism may change over time as a country develops.

However inequality has a definite negative effect at early stages of development. The reason is that at these stages, the *growth value* of  $g(\gamma_t)$  (which is given by  $\sum_i \beta_i z_{it}$ ; see the second right-hand term of expression [6.8]) tends to be null because the preference for quality at low income levels is very low (note that  $x_{it}$  and  $z_{it}$  are close to zero when  $A_t$  is close to zero). In other words, being able to produce higher quality at low cost has little value at early stages of development. Moreover  $g(\gamma_t)$  in the last term of (6.8) is also close to zero since  $x_{it}$  and  $z_{it}$  are close to zero. So only the negative effect of inequality on  $g(A_t)$  is quantitatively relevant at early stages. Conversely at more advanced stages of development, the effect on  $g(A_t)$  becomes small whereas the value of raising quality efficiency  $\gamma_t$  increases. Therefore the sign of the inequality/growth relationship may change to positive depending on the specific value of the parameters.

**Proposition 6.1.** Inequality has a negative impact on growth at early stages of development through the quantity/quality mechanism.

#### **Proof.** See appendix.

The intuitive argument for this proposition is as follows. More unequal poor economies spend a larger share of GDP in the production of luxury goods consumed by the elite. The resulting induced learning has limited possibilities of generating important productivity gains for two reasons.

First, achieving significant learning/technical progress requires a large scale (quantity) of production. However producing for the elite in a poor country implies that the scale of production is small. And second, technical progress in producing higher-quality goods may be useless for producing the varieties consumed by the majority of the population (who, even in poor unequal societies, absorb a large share of GDP).

As pointed out in the Introduction, the prediction in this proposition is consistent with the most empirical evidence on the relationship between inequality and growth. Of special relevance for this working paper is the evidence in Barro (2000) and Barro (2008) who controls most mechanisms previously suggested in the literature and still finds a significant nonlinear impact of inequality and growth, as predicted by our model.

# 7. Concluding Comments

 $m extbf{K}$ ECENT empirical work in several areas such as real gross domestic product (GDP) growth measurement and international trade has uncovered the increasing importance of the quality dimension of output. This working paper explores the consumer time-constraint foundations of the demand for quality and some of its macroeconomic implications. The starting point is that time and consumption are complements, and that higher-quality goods provide higher utility per unit of time though at a higher monetary cost. It is then shown that additions to income are increasingly spent upgrading the quality of consumption. The working paper provides a tractable framework where macroeconomic issues related to the quality dimension of goods can be analyzed. Four topics receive a first-bite examination: the increasing importance of quality growth as a component of GDP growth, the quality bias of technical progress, the impact of inequality on growth through the quantity/quality mechanism, and the role of progressive consumption taxes as a growth policy. In spite of its simple structure, the endogenous growth model in our working paper can deliver reasonable quantitative paths for secular GDP in advanced economies. However the calibration put forward seems to largely overestimate the quality component of GDP growth. Building and calibrating models that can match the quantity and quality components of long-run GDP growth—as more thorough estimates of these components become available—and that can be linked to the recent patterns of international trade, technical progress, and the demand for skilled labor, seems an interesting area for future research. Models embedding the increasing importance of quality growth may also provide new insights for growth policy.

# Appendix: Proofs of Propositions

#### **Proof of proposition 5.1**

To check this proposition, first note that  $A^*>0$ ,  $x^*>0$ , and  $(q/\gamma)^*>0$  imply  $g_A^*=g_x^*=g_z^*=0$ ,  $z^*>0$ ,  $x^*>0$ . That is, in the steady state there is constant positive gross domestic product (GDP) growth due to quality upgrading but no quantitative growth. Moreover all technical progress comes from increases in quality-biased efficiency  $\gamma$ . Substituting with  $g_A^*=0$  in (5.10) yields  $z^*\equiv (q/\gamma)^*=\ln\frac{\theta_1}{\delta_1}>0$ . In turn, expressions (5.7) and (5.8) yield  $x^*=\sigma\frac{\ln{(\theta_1/\delta_1)}}{1+\ln{(\theta_1/\delta_1)}}$ ;  $A^*=x^*\frac{\theta_1}{\delta_1}$ . Then,  $g(z_t)=g(q_t)-g(\gamma_t)$  and (5.11) imply  $g_q^*=g_\gamma^*=\theta_2z^*x^*-\delta_2$ . Finally (5.9) yields:

$$g_{y}^{*} = z^{*}g_{\gamma}^{*} = \ln \left(\frac{\theta_{1}}{\delta_{1}}\right). \left[\theta_{2}\sigma \frac{(\ln (\theta_{1}/\delta_{1}))^{2}}{1 + \ln (\theta_{1}/\delta_{1})} - \delta_{2}\right].$$

#### **Proof of proposition 6.1**

Early stages of development are characterized by low levels of general efficiency A. Furthermore increases in inequality are defined as redistributions of wealth from individuals with lower  $\beta_i$  to individuals with higher  $\beta_i$ . Thus to prove proposition 6.1, we will show that for A sufficiently small,  $d^2 g(Y_t)/d\beta_i^2$  in expression (6.7) is negative.

Before turning to that derivative, first consider the derivatives of  $x_{ib}$   $z_{ib}$  and  $x_{il}z_{il}$  with respect to  $\beta_i$ . The time subscript does not play any role in the following and therefore we suppress it. Given A at some point in time,  $x_i$  and  $z_i$  can be solved from the system (5.8)-(6.6). Using this system (and taking into account that in equilibrium  $x_i = \sigma \frac{z_i}{1+z_i}$ ), the derivatives of  $x_i$  and  $z_i$  with respect to  $\beta_i$  yield:

$$\frac{dx_i}{d\beta_i} = \frac{A}{e^{z_i}} \frac{(\sigma - x_i)^2}{(\sigma - x_i)^2 + \sigma x_i} > 0, \quad \frac{d^2x_i}{d\beta_i^2} = -\frac{A}{e^{z_i}} \frac{(z_i + z_i^2 + 1)(z_i + 2)(z_i + 1)\sigma^3}{(\sigma + z_i\sigma + z_i^2)^3} \frac{dz_i}{d\beta_i} < 0,$$

$$\frac{dz_i}{d\beta_i} = \frac{A}{e^{z_i}} \frac{(z_i + 1)^2}{(z_i + z_i^2 + 1)\sigma} > 0, \quad \frac{d^2z_i}{d\beta_i^2} = -\frac{A}{e^{z_i}} \frac{(2z_i + z_i^2 + 3)(z_i + 1)dz_i}{(z_i + z_i^2 + 1)^2\sigma} \frac{d\beta_i}{d\beta_i} > 0,$$

$$\frac{d(x_i z_i)}{d\beta_i} = \sigma \frac{(z_i + 2) z_i}{(z+1)^2} \frac{dz_i}{d\beta_i} > 0, \quad \frac{d^2(x_i z_i)}{d\beta_i^2} = -\frac{A}{e^{z_i}} \frac{(z_i^3 + 2 (z_i^2 + z_i - 1))(z_i + 1)}{(z_i + z_i^2 + 1)^2} \frac{dz_i}{d\beta_i}.$$

The sign of the last expression depends on the size of  $z_i$ . Now consider the second derivative of expression (6.7) with respect to  $\beta_i$ :

$$\frac{d^2 g(Y_t)}{d\beta_i^2} = \frac{d^2 g(A)}{d\beta_i^2} + \frac{d^2 g(\gamma)}{d\beta_i^2} \sum_i \beta_i z_i + 2 \frac{dg(\gamma)}{d\beta_i} \left[ \beta_i \frac{dz_i}{d\beta_i} + z_i \right] + g(\gamma) \left[ \beta_i \frac{d^2 z_i}{d\beta_i^2} + 2 \frac{dz_i}{d\beta_i} \right] = H \frac{dz_i}{d\beta_i},$$

where

$$H = -\theta_1 \frac{1}{e^{z_i}} \frac{(z_i + z_i^2 + 1) (z_i + 2) (z_i + 1)\sigma^3}{(\sigma + z_i\sigma + z_i^2)^3} + \theta_2 \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i^2) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^3}{(1 + z_i + z_i^2)^2} (1 + z_i^2) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^2}{(1 + z_i + z_i^2)^2} (1 + z_i^2) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^2}{(1 + z_i^2 - z_i)^2} (1 + z_i^2) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^2}{(1 + z_i^2 - z_i)^2} (1 + z_i^2) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^2}{(1 + z_i^2 - z_i)^2} (1 + z_i^2) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^2}{(1 + z_i^2 - z_i)^2} (1 + z_i^2 - z_i) \sum_i \beta_i z_i + \frac{A}{e^{z_i}} \frac{2(1 - z_i^2 - z_i) - z_i^2}{(1 + z_i^2 - z_i$$

$$+2\theta_{2}\sigma z_{i}\frac{2+z_{i}}{(1+z_{i})^{2}}\left[\beta_{i}\frac{dz_{i}}{d\beta_{i}}+z_{i}\right]+g\left(\gamma\right)\left[\beta_{i}\frac{A}{e^{z_{i}}}\frac{2\left(1-z_{i}^{2}-z_{i}\right)-z_{i}^{3}}{\left(1+z_{i}+z_{i}^{2}\right)^{2}}\left(1+z_{i}\right)+2\right].$$

Taking into account that  $\lim_{A\to 0} = \lim_{A\to 0} z_i = 0$ , we have

$$\lim_{A \to 0} H = -2 (\theta_1 + \delta_2) < 0.$$

Note that  $x_i$ ,  $z_i$  and all the derivatives above are continuous in A. Therefore for A sufficiently close to 0 we have  $d^2g(Y_t)/d\beta_i^2 < 0$ .

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#### $A\ B\ O\ U\ T$ $T\ H\ E$ $A\ U\ T\ H\ O\ R\ *$

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